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TITLE: The divided cell algorithm and the inhomogeneous Lagrange and Markoff spectra

ABSTRACT: In the 1950s E. S. Barnes and H. P. F. Swinnerton-Dyer developed the divided cell algorithm to calculate the inhomogeneous minima of binary quadratic forms; that is, elements of inhomogeneous Markoff spectra. Their restriction to nonzero forms is not necessary, which means the process can also be applied to inhomogeneous approximation of real numbers; that is, to calculate elements of inhomogeneous Lagrange spectra. In this talk we show how advances of the past fifty years in both symbolic computation and our understanding of homogeneous spectra can be combined to make the divided cell algorithm more computationally feasible. In fact, we will present some important examples which can be easily computed by hand.

A crucial part of our analysis relies on work of Jane Pitman, who related the divided cell algorithm to the regular continued fraction algorithm. The connection can best be described in terms of the lattice interpretation of the continued fraction as a process which yields a two-sided convergent sequence containing all points for which the absolute value of the (homogeneous) form is small. In addition, the process identifies the important convergents in terms of large partial quotients. Analogously, the chain of divided cells for all inhomogeneous problems based on a fixed quadratic form can be described in terms of the continued fractions of the roots of the form. Further, every point which gives a small value of the inhomogeneous form is a vertex of the two-sided sequence of divided cells. Just as the theory of continued fractions identified the important convergents, the theory of divided cells can be used to identify the important vertices for the inhomogeneous problems.